Mechanics, Heat, Oscillations and Waves - Assignments

The assignments have been framed so that a student can attempt assignment(s) number N and N(b) after having watched the video lectures of week N.

In the True/False type questions, please state whether the statement within quotes is true or false. If there is anything outside the quotes, that is just extra information to set up the scenario. Answers are given in bold and enclosed in square brackets []. Answers to questions involving vector quantities are not given in bold since vectors are themselves typed in boldface.

Assignment 1

A. TRUE/FALSE

1. "From c (speed of light), $\hbar$ (Planck's constant divided by 2π) and G (gravitational constant), unique combinations with the dimensions of time (T), length (L) and mass (M) can be formed." [**TRUE**]

2. "From dimensional analysis, the time period of a simple pendulum can be deduced exactly." [**FALSE**]

3. "The function $f(x) = e^{-x}$ cannot be negative for any real value of x." [**TRUE**]

4. "Of the four fundamental forces of nature, only gravitation has a range greater than 10^{-10}m." [**FALSE**]

5. "The weak force is weaker than the electromagnetic force, but stronger than gravitation." [**TRUE**]

B. Fill in the blanks

6. The physical dimensions of the coefficient of viscosity of a liquid are, $[\eta]$ = ___. [$\mathbf{ML^{-1}T^{-1}}$]

7. The force on a particle moving in space is given by $\mathbf{F} = -dV(r)/dr\ \mathbf{e}_r$, where $\mathbf{e}_r$ is the unit vector in the radial direction. The physical dimensions of V(r) are ___. [$\mathbf{ML^{2}T^{-2}}$]

Assignment 1(b)

Sketch the following functions. (These are commonly used functions and you can easily verify your answer from the internet.)

1. e^{-x}, $-\infty < x < \infty$

2. $1-\cos(\theta)$, $-\infty < \theta < \infty$

3. $1/(1+x^2)$, $-\infty < x < \infty$

4. $\ln(x)$, $0 < x < \infty$

5. x^x, $0 \le x < \infty$

6. $(x-2)/(x-1)$, $-\infty < x < \infty$

7. $\tanh(x)$, $-\infty < x < \infty$

8. $e^{(-1/x)}$, $-\infty < x < \infty$

9. $|\sin(x)|$, $-\infty < x < \infty$

10. $\sin(x)/x$, $-\infty < x < \infty$

Assignment 2

A. TRUE/FALSE

1. Let (ρ,ϕ) denote the plane polar coordinates in the (x,y) plane. The ranges of ρ and ϕ are, respectively, $0 \le \rho < \infty$ and $0 \le \phi < 2\pi$.
   "Given (x,y), the polar coordinates (ρ,ϕ) are uniquely determined for all points except the origin in the (x,y) plane." [**TRUE**]

2. "A scalar is a quantity that remains unchanged under a rotation of the coordinate axes." [**TRUE**]

3. "A vector in 3-dimensions is an ordered set of 3 quantities (u_1,u_2,u_3) that transform under a rotation of the coordinate axes in exactly the same way as the coordinates (x,y,z) themselves transform." [**TRUE**]

4. Let $\mathbf{v} = (v_1,v_2,v_3)$ be a vector. "The quantity $(v_1^2+v_2^2+v_3^2)$ is a scalar." [**TRUE**]

5. Let $\mathbf{v} = (v_1,v_2,v_3)$ be a vector. "Each of the quantities $|v_1|,|v_2|,|v_3|$ is a scalar." [**FALSE**]

6. "From two vectors **a** and **b**, the only independent scalars that can be constructed are $\mathbf{a}\cdot\mathbf{a}$, $\mathbf{a}\cdot\mathbf{b}$ and $\mathbf{b}\cdot\mathbf{b}$." [**TRUE**]

7. "If **a** and **b** are polar vectors, then $\mathbf{a}\otimes\mathbf{b}$ is an axial vector." (Here, $\otimes$ denotes cross product) [**TRUE**]

8. "If **a**,**b**,**c** are polar vectors, then $\mathbf{a}\cdot(\mathbf{b}\otimes\mathbf{c})$ is a pseudoscalar." [**TRUE**]

9. Let **a**,**b**,**c** be three arbitrary, non-coplanar vectors in 3-dimensional space.
   "Any vector **u** can be written as a linear combination $\mathbf{u} = \alpha\mathbf{a}+\beta\mathbf{b}+\gamma\mathbf{c}$." [**TRUE**]

B. Answer the following questions

10. If (ρ,ϕ) are the plane polar coordinates of a point in the (x,y) plane, the unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are given in terms of the unit vectors $\mathbf{e}_\rho$ and $\mathbf{e}_\phi$ by $\mathbf{e}_x = \alpha\mathbf{e}_\rho + \beta\mathbf{e}_\phi$ and $\mathbf{e}_y = \gamma\mathbf{e}_\rho + \delta\mathbf{e}_\phi$.
    The coefficients α, β, γ and δ are (respectively) ___.   [**cos(ϕ) -sin(ϕ) sin(ϕ) cos(ϕ)**]

11. <u>Parity transformations</u>: Under a coordinate transformation x→x'=-x,y→y'=-y,z→z'=-z, the transformation properties of the quantities listed below are (*Write INV if you think the given quantity does not change (invariant) under the given parity transformation, and NEG if you think it gets a negative sign upon transformation, and ELSE if you think something other than the two cases mentioned above happens*):
    v = [**NEG**]
    p = [**NEG**]
    r·p = [**INV**]
    L = **r x p** = [**INV**]
    E = [**NEG**]
    B = [**INV**]
    E·B = [**NEG**]
    E x B = [**NEG**]

12. Let (ρ,ϕ) denote the plane polar coordinates in the (x,y) plane. A particle follows the paths specified below (in Cartesian coordinates). In each case, express the path in plane polar coordinates.
    (i) $x^2+y^2=a^2$ ('a' is a positive constant)   [$\boldsymbol{\rho = a}$]
    (ii) y=mx (x,y ≥ 0; m=positive constant)   [$\boldsymbol{\phi = \tan^{-1}(m)}$]

13. Let (ρ,ϕ) denote the plane polar coordinates in the (x,y) plane. A particle follows the paths specified below (in polar coordinates). In each case, express the path in Cartesian coordinates.
    (i) $\rho=1/\sqrt{(\cos(2\phi))}$ ($0 \le \phi < \pi/4$)   [$\mathbf{x^2-y^2=1}$]
    (ii) $\rho=1/\sqrt{|\cos(2\phi)|}$ ($\pi/4 < \phi < 3\pi/4$)   [$\mathbf{y^2-x^2=1}$]

Assignment 3

A. TRUE/FALSE

1. Let (ρ,ϕ) be plane polar coordinates and $\mathbf{e}_\rho$, $\mathbf{e}_\phi$ be the corresponding unit vectors. "Then $\mathbf{e}_\rho$ and $\mathbf{e}_\phi$ are functions of ρ, but not of ϕ." [**FALSE**]

2. "The unit vectors $(\mathbf{e}_r,\mathbf{e}_\theta,\mathbf{e}_\phi)$ in spherical polar coordinates form a right handed triad, i.e., $\mathbf{e}_r\otimes\mathbf{e}_\theta=\mathbf{e}_\phi$, $\mathbf{e}_\theta\otimes\mathbf{e}_\phi=\mathbf{e}_r$, $\mathbf{e}_\phi\otimes\mathbf{e}_r=\mathbf{e}_\theta$." [**TRUE**]

3. "Under an arbitrary rotation of the coordinate axes about the origin in 3-dimensional space, any vector **A** transforms to **A'**, such that $A^2=A'^2$ ". [**TRUE**]

4. "The position vector of any point in spherical polar coordinates is given by $\mathbf{r}=r\mathbf{e}_r+\theta\mathbf{e}_\theta+\phi\mathbf{e}_\phi$." [**FALSE**]

5. "An arbitrary infinitesimal displacement from **r** to **r**+d**r** is given, in spherical polar coordinates, by $d\mathbf{r}=(dr)\mathbf{e}_r+(rd\theta)\mathbf{e}_\theta+(r\sin\theta d\phi)\mathbf{e}_\phi$." [**TRUE**]

6. "Uniform motion of a particle in a circular path requires a centripetal force." [**TRUE**]

7. "If the angular speed of a particle moving in a circle increases with time, the particle must necessarily be acted upon by a tangential force." [**TRUE**]

8. "Newton's 1st law of motion is a special case of the 2nd law, and can be derived from it." [**FALSE**]

9. "Newton's 3rd law of motion can be derived from the 2nd law." [**FALSE**]

10. "In Newton's 3rd law of motion, action and reaction always act on different bodies." [**TRUE**]

11. "When a point mass m moves in a circle with uniform angular speed, its orbital angular momentum about the centre remains constant in time." [**TRUE**]

12. A system of N particles interact with each other pairwise, i.e., each particle exerts a force on every other particle. The force between the particles is directed along the line joining them. "The sum $\mathbf{p}_i+\mathbf{p}_j$ of the momenta of every pair of particles *i* and *j*remains constant in time." [**FALSE**]

B. Fill in the blanks

13. **a**,**b** and **c** are three non-coplanar vectors in three-dimensional space. The volume of the parallelopiped formed by **a**,**b** and **c** is _____. [$|\mathbf{a}\cdot(\mathbf{b}\otimes\mathbf{c})|$, $|\mathbf{c}\cdot(\mathbf{a}\otimes\mathbf{b})|$]

14. In a coordinate frame S, the force on a particle is given by **F** (F_x, $F_y$and F_z are the three components). Consider a frame S' obtained by rotating S about the y-axis through an angle $\pi/4$, keeping the origin unchanged. In S', the force on the particle is $\mathbf{F'}$=__$\mathbf{e_x}$+__$\mathbf{e_y}$+__$\mathbf{e_z}$
[$(F_x-F_z)/\sqrt{2}$, F_y, $(F_x+F_z)/\sqrt{2}$]

15. A particle moves in the x-y plane in a circular path of radius R centered at the origin. Its angular velocity $\omega=d\phi/dt$ varies with time. Its linear speed v is given by v=___ [ωR]

16. A particle moves on the circle $x^2+y^2=R^2$. In terms of the plane polar coordinates ρ and ϕ (and the corresponding unit vectors $\mathbf{e}_\rho$ and $\mathbf{e}_\phi$), its acceleration is given by **a**=___.
[$R(d^2\phi/dt^2)\mathbf{e}_\phi$ - $R(d\phi/dt)^2\mathbf{e}_\rho$]

17. Under a rotation R(**n**,ψ) (where **n** is the unit normal vector) of the coordinate axes about the origin, a vector $\mathbf{A}\rightarrow\mathbf{A'}$. Under a rotation R(-**n**,ψ), let the vector $\mathbf{A}\rightarrow\mathbf{A''}$.
Then, in terms of **A**, **n** and ψ, the difference $\mathbf{A'}-\mathbf{A''}$ = ___. [$2\sin\psi\ (\mathbf{A}\otimes\mathbf{n})$]

18. The total kinetic energy of two particles of masses m_1 and m_2, and momenta $\mathbf{p_1}$ and $\mathbf{p_2}$, is given by $T=\mathbf{p_1}^2/(2m_1)+\mathbf{p_2}^2/(2m_2)$. Define $\mathbf{P}=\mathbf{p_1}+\mathbf{p_2}$ and $\mathbf{p}=(m_2\mathbf{p_1}-m_1\mathbf{p_2})/(m_1+m_2)$.
If T can be expressed as $T=\mathbf{P}^2/(2M)+\mathbf{p}^2/(2\mu)$, then M=___ and μ=___.
[m_1+m_2, $m_1m_2/(m_1+m_2)$]

Assignment 3(b)

Answer the questions below based on the given scenario.

A rocket moves in a planar path that is in the shape of an outward spiral. Let the path lie in the x-y plane, and let a, b, ω be positive constants. The rocket starts from the point ρ=a, ϕ=0 in polar coordinates (or x=a, y=0 in Cartesian coordinates) and moves along the curve $\rho = ae^{b\phi}$, where $\phi = \omega t$.

1. What is the instantaneous velocity **v**(t) of the rocket? [$a\omega e^{b\omega t}(b\mathbf{e}_\rho+\mathbf{e}_\phi)$]

2. What is the speed of the rocket at any time t? [$\boldsymbol{a\omega e^{b\omega t}\,(b^2+1)^{1/2}}$]

3. What is the acceleration **a**(t) of the rocket at time t? [$a\omega^2(b^2-1)e^{b\omega t}\mathbf{e}_\rho + 2ab\omega^2 e^{b\omega t}\mathbf{e}_\phi$]

4. The path of the rocket intersects any half-line [ϕ = constant] at the same angle α. What is α? [$\boldsymbol{\alpha = \cot^{-1}(b)}$]

5. What is the value of α when the acceleration of the rocket is purely tangential? [$\boldsymbol{\pi/4}$]

Assignment 4

A. TRUE/FALSE

1. "In a two-body elastic collision, knowledge of the initial momenta $\mathbf{p_1}$and $\mathbf{p_2}$ is sufficient to determine the final momenta $\mathbf{p_1'}$ and $\mathbf{p_2'}$uniquely, using the conservation of energy and conservation of momentum." [**FALSE**]

2. "The electrostatic force on a test charge due to a configuration of fixed charges is a conservative force." [**TRUE**]

3. A particle of mass m collides elastically with a stationary target of equal mass. "The projectile can never be scattered through an angle greater than 45^o with respect to its original direction of motion." [**TRUE**]

B. The next 8 questions are based on the scenario set up below. Fill in the blanks.

A particle of mass m_1 has an initial momentum $\mathbf{p_1}=p_1\mathbf{e_x}$ in the lab frame of reference. It collides elastically with a particle of mass $m_2$that is initially at rest. The momenta of the particles after the collision are given, respectively, by

$\mathbf{p_1'} = p_1'\cos(\alpha)\mathbf{e_x} + p_1'\sin(\alpha)\mathbf{e_y}$, and

$\mathbf{p_2'} = p_2'\cos(\beta)\mathbf{e_x} - p_2'\sin(\beta)\mathbf{e_y}$

Here $p_1' = p_1(1-2r\cos(2\beta)+r^2)^{1/2}/(1+r)$, and $p_2' = 2p_1\cos(\beta)/(1+r)$ and $r=m_1/m_2$.

Further, α and β are related to each other according to: $\tan(\alpha) = \sin(2\beta)/(r-\cos(2\beta))$.

4. When r=1, $\alpha + \beta$ = ___. [$\boldsymbol{\pi/2}$]

5. For any r, the maximum possible value of β is ___. [$\boldsymbol{\pi/2}$]

6. When r>1, the largest possible value of α is ___. [$\mathbf{\sin^{-1}(m_2/m_1)}$]

7. The initial kinetic energy of the projectile is $E_1 = p_1^2/(2m_1)$. The energy transferred to the target is $E_2 = p_2^2/(2m_2)$.
   The ratio of these energies is, in terms of r and β, is E_2/E_1 = ____. [$\mathbf{4r\cos^2(\beta)/(1+r)^2}$]

8. It is possible to observe the collision from a frame of reference (called the Centre of Momentum frame) that is moving at a constant velocity **v** with respect to the lab frame such that the initial momenta of the masses m_1 and m_2 in this frame are equal in magnitude and opposite in direction, i.e., given by **p** and **-p**, respectively.
   This velocity **v** is given by $\mathbf{v} = \mathbf{p_1}/M$. What is M equal to? [$\mathbf{m_1 + m_2}$]

9. In the Centre of Momentum frame, $\mathbf{p}$ = ___. [$(m_2\mathbf{p_1})/(m_1+m_2)$]

10. After the collision, the final momenta of the masses $m_1$and m_2 are again equal in magnitude and opposite in sign, **p'** and **-p'**, say. Then, if $\mathbf{-p'} = p_1[\cos(\phi)\mathbf{e_x}-\sin(\phi)\mathbf{e_y}]/(1+r)$, what is ϕ? [$\mathbf{2\beta}$]

11. The scattering angles of m_1 and m_2 in the centre of momentum frame are θ and π-θ, respectively. θ and β are related by ____. [$\mathbf{2\beta = \pi-\theta}$]

Assignment 5

A. TRUE/FALSE

1. A point mass m is fixed to each of the vertices of an equilateral triangle whose centre is O. "The gravitational potential at O is zero." [**FALSE**]

2. Let $\phi(\mathbf{r})$ be the potential associated with a conservative force $\mathbf{F}(\mathbf{r})$. "$\mathbf{F}(\mathbf{r})$ must be zero at all points at which $\phi(\mathbf{r})$ is zero, but the converse is not necessarily true." [**FALSE**]

3. "Kepler's 2nd law continues to remain valid for a mass moving under the force field $+K\mathbf{r}/r^3$ (K>0) in a hyperbolic trajectory." [**TRUE**]

4. "Among central forces, only the forces $\mathbf{F}=(-K/r^2)\mathbf{e}_r$ and $\mathbf{F}=-K\mathbf{r}$ (where K is a positive constant of appropriate dimensions in each case) lead to closed orbits for every bounded motion." [**TRUE**]

B. Answer the next 8 questions are based on the scenario described below.

The potential energy of a particle of mass m moving on the x-axis is given by $V(x)=kx(x^2-3a^2)$, where k and a are positive constants of appropriate physical dimensions.

5. Find the values of x at which V(x) has a maximum or a minimum. (x_{max} and x_{min} are the points of maxima and minima, respectively.) [$\mathbf{x_{max} = -a, x_{min} = a}$]

6. What are the values of the potential at these points of minima and maxima? (V_{min} and V_{max}, respectively) [$\mathbf{V_{min}= -2ka^3, V_{max}= 2ka^3}$]

7. What is the force, F(x), on the particle? (Since the system is one-dimensional, ignore the fact that the force is a vector. Consider it to be a scalar.) [$\mathbf{F(x) = 3k(a^2 - x^2)}$]

8. What is the shape of the graph of the function F(x)? [**Parabolic**]

9. Suppose the particle is released with a negligible velocity from the point x=-a at t=0. What is the largest value of x that the particle reaches before turning back?
   (Hint: It may be useful to draw the graph of V(x) to help you to analyze this process.) [**x=2a**]

10. Using dimensional analysis, determine which of the following quantities can be a possible measure of the time period of the motion described in the previous question (time taken to go from x=-a to the maximum point and return to x=-a). [$\mathbf{(m/(ka))^{1/2}}$]

11. If the time taken by the particle to return to its starting point x = -a can be expressed as $T = [pm/(ka)]^{1/2} \int_{-1}^{2} du/\sqrt{[2+3u-u^3]}$, where p is a positive integer, what is the value of p? [**2**]

12. What is the maximum value of the kinetic energy of the particle during the round trip from x = -a back to this point? [$\mathbf{2ka^3}$]

Assignment 6

A. TRUE/FALSE

1. The instantaneous position of a simple harmonic oscillator is given by $x(t)=A\cos(\omega t+\delta)$, where A and δ are positive constants and $0<\delta<\pi/2$.
   "The total energy of the oscillator is independent of δ." [**TRUE**]

2. The instantaneous position of a simple harmonic oscillator is given by $x(t)=A\sin(\omega t+\delta)$, where A and δ are positive constants and $0<\delta<\pi/2$.
   "The total energy of the oscillator is independent of δ." [**TRUE**]

3. "The restoring force acting on a simple harmonic oscillator is a conservative force." [**TRUE**]

B. Answer the next 5 questions are based on the following scenario.

A ring of radius R is fixed in the x-y plane with its centre at the origin O. The ring has a uniform positive charge density λ per unit length. A particle of mass m and charge -q is released from rest from the point (0,0,a) on the z-axis at t=0.

4. What is the instantaneous force on the particle at t=0? [$-qR\lambda a/\{2\varepsilon_0(a^2+R^2)^{3/2}\}\ \mathbf{e_z}$]

5. What is the instantaneous force on the particle at any time t>0, when its coordinates are (x,y,z)? [$-qR\lambda z/\{2\varepsilon_0(z^2+R^2)^{3/2}\}\ \mathbf{e_z}$]

6. What path does the particle follow for t>0? [Moves along z-axis, passing through origin]

7. What is the potential energy V(z) of the particle? [$\boldsymbol{-qR\lambda/\{2\varepsilon_0(z^2+R^2)^{1/2}\}}$]

8. When a<<R, the particle executes simple harmonic motion on the z-axis about the origin O. What is the time period of this simple harmonic oscillation? [$\boldsymbol{2\pi R\,\{2\varepsilon_0 m/(q\lambda)\}^{1/2}}$]

Assignment 7

A. Fill in the blanks

1. A solid cube is made of a material of Young's modulus E and Poisson's ratio ν. A uniaxial tension σ is applied along on the principal axes of the cube. In terms of E, ν and σ, the volumetric strain of the cube is, ΔV/V = ___. [**(1-2ν)σ/E**]

2. A particle undergoes uniform circular motion at a speed v on the circle $x^2+y^2=R^2$. Its y-coordinate therefore executes simple harmonic motion with amplitude = ___ and time period = ___. [**R** and **2πR/v**]

3. A cylindrical rod of length l, cross sectional area A and Young's modulus E, is stretched to a length l+Δl by a uniaxial stress along its axis. Neglect the lateral contraction of the rod. The energy of elastic deformation stored in the rod is ___. [**$EA(\Delta l)^2/(2l)$**]

4. A spring of spring constant k (and negligible mass) hangs vertically, its upper end being attached to a support. Its lower end is at the point P. A mass m is now attached to the lower end. The equilibrium position of the lower end of the spring is now at a point labelled as Q. The distance PQ = ___. When the mass is pulled down slightly and released, it executes simple harmonic motion with a time period T = ___. [**PQ = mg/k** and **$T = 2\pi(m/k)^{1/2}$**]

5. A particle of mass m can execute simple harmonic motion in the x and y directions in the x-y plane. Its instantaneous position coordinates are given by x(t) = a cos(ωt), y(t) = b sin(ωt+δ), where a, b and δ are positive constants. Its trajectory in the x-y plane is given by the curve whose equation is ___. The shape of the curve is ___. [**$(x^2/a^2) + (y^2/b^2) - 2xy\sin(\delta)/(ab) = \cos^2(\delta)$; An ellipse**]

6. A rope of length L and uniform linear mass density (mass per unit length) ρ hangs down from a hook in the ceiling of a room. The vertical coordinate of its lower end is z=0, and that of its upper end is z=L. A transverse pulse is set up in the rope by shaking its lower end. The time taken for the pulse to travel up the rope to the ceiling and get reflected back to z=0 is T = ___. [**$4(L/g)^{1/2}$**]

7. The waveform $\phi(x,t) = A \sin(\omega t-kx)$ satisfies the wave equation $\partial^2\phi/\partial t^2 = v^2 (\partial^2\phi/\partial x^2)$, provided the condition ___ is satisfied. [$\boldsymbol{\omega/k = v}$]

8. A siren that emits sound at 3000Hz is mounted on a car. The car moves towards an observer at a speed v/2. The observer, who is in another car, also moves towards the siren at a speed v/2. The observer hears a sound of frequency 3600Hz. Given that the speed of sound is 330m/s, the relative speed of one car with respect to the other car is ___km/h. [**216**]

9. According to wave-particle duality (in quantum mechanics), the relation between the frequency and wavenumber of matter waves corresponding to a freely moving particle of mass m is given by $\omega = \hbar k^2/(2m)$, where $\hbar$ is a positive constant (it is Planck's constant divided by 2π). Hence, the ratio of the group velocity of these waves to their phase velocity is ___. [**2**]

10. The product of the wave and group velocities of certain waves is found to be a constant, α. The angular frequency $\omega(k)$ is a function of the wavenumber, k. Further, it is given that $\omega(k=0) = k_0\sqrt{\alpha}$, where is k_0 is a positive constant. Hence, the relation between ω and k for these waves is given by ω^2 = ___. [$\boldsymbol{\alpha(k^2+k_0^2)}$]

11. A compressible gas undergoes steady, streamlined flow. Its pressure (P) and density (ρ) are related according to P/ρ^γ = constant. (γ is a positive constant. It is the ratio of specific heats at constant pressure and constant volume.) There is no external force on the gas. Then, according to Bernoulli's principle, what is the function of P, ρ and the velocity (magnitude is v), what remains constant on a streamline? [$\boldsymbol{\{\gamma/(\gamma-1)\}(P/\rho) + v^2/2 = \text{constant}}$]

B. Answer the next 3 questions based on the following scenario.

The velocity field of a fluid undergoing steady streamlined flow in a cylinder is given, in cylindrical polar coordinates, by $\mathbf{v}(\rho,\phi,z) = \nabla\phi$, where $\rho=(x^2+y^2)^{1/2}$ and $\phi=\tan^{-1}(y/x)$ as usual.

12. In Cartesian coordinates, the velocity field is given by $\mathbf{v}(x,y,z)$ = ___. [$(-y\mathbf{e}_x+x\mathbf{e}_y)/(x^2+y^2)$]

13. In terms of the unit vectors in cylindrical polar coordinates, the velocity field is given by $\mathbf{v}(\rho,\phi,z)$ = ___. [$\mathbf{e}_\phi/\rho$]

14. The streamlines are curves given in cylindrical polar coordinates by ___. [**Concentric circles (ρ=constant, z=constant) about the z-axis, traversed in the positive sense**]

Assignment 8

A. Fill in the blanks

1. In the absence of sources and sinks, the equation of continuity for fluid flow is ___. When the fluid is incompressible, this reduces to ___. [$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$; $\nabla\cdot\mathbf{v} = 0$]

2. Consider the steady, streamlined flow of water (of constant density ρ) in a uniform pipe of length L. The inlet of the pipe is at a height z = h above the ground, while the outlet is at ground level, z = 0. The fluid pressure at the inlet is P_1, and that at the outlet is the atmospheric pressure P_0. Assume that the inlet speed v_1 of the water is negligible. The amount by which the outlet speed v_0 exceeds what it would have been under free fall through a height h is equal to ___. [$\mathbf{\{2gh + 2(P_1-P_0)/\rho\}^{1/2} - (2gh)^{1/2}}$]

3. Consider the steady, streamlined flow of a compressible fluid. Assume that the flow is barotropic, i.e., the density of the fluid is a given function of its pressure, ρ = ρ(P). There is no external force on the fluid. According to Bernoulli's Principle, the quantity that remains constant on each streamline is ___. [$\mathbf{\int dP/\rho(P) + v^2/2}$]

4. The pressure P of n moles of an ideal gas is given by the equation of state PV = nRT, where R is the gas constant. In terms of the intensive quantities ρ (the mass per unit volume of the gas) and T, the equation of state is P = ___.
[**$\rho k_B T/m$, where m is the mass of a molecule and k_B is Boltzmann's constant.**]

5. The Van der Waals equation of state for one mole of a gas is $P = RT/(V-b) - a/V^2$, where a and b are positive constants. The critical volume V_c, critical temperature T_c and critical pressure P_c are found by imposing the additional conditions $(\partial P/\partial V)_T = 0$ and $(\partial^2P/\partial V^2)_T = 0$, along with the equation of state. (The critical point corresponds to a point on the critical isotherm at which the slope and curvature of the isotherm in the (V,P) plane vanish.)
The solutions are V_c = ___, T_c = ___, P_c = ___. [$\mathbf{V_c = 3b,\ T_c = 8a/(27Rb),\ P_c = a/(27b^2)}$]

6. *Continued:* The so-called reduced volume, reduced temperature and reduced pressure are defined as the following dimensionless quantities $v = (V-V_c)/V_c$, $t = (T-T_c)/T_c$ and $p = (P-P_c)/P_c$. Using these variables, the Van der Waals equation of state can be written in a form in which the constants a, b and R do not appear explicitly. This form is p = ___.
[$\mathbf{8(1+t)/(3v+2) - 3/(1+v)^2 - 1}$]

7. *Continued:* Hence, for small v, the leading behaviour of p as a function of v on the critical isotherm (which corresponds to setting $T = T_c$, or t = 0) is given by p = ___. **[$p \cong -(3/2)v^3$]**
{Hint: Set t = 0. Use the binomial theorem to expand the factors $(1 + 3v/2)^{-1}$ and $(1+v)^{-2}$ in powers of v. Note that the terms proportional to v^0, v^1 and v^2 all cancel out.}

8. Consider a given quantity of a single-component fluid. When the independent thermodynamic variables are its entropy S and volume V, the relevant thermodynamic potential is the internal energy U = U(S,V). The laws of thermodynamics then state that the differential dU = ___. {In thermal equilibrium, U is at a minimum.} [**TdS − PdV**]

9. *Continued:* If, on the other hand, the temperature T and volume V are taken to be the independent thermodynamic variables, the appropriate thermodynamic potential (F) that is at a minimum in thermal equilibrium is ___. Its differential is given by ___.
[**F ≡ U − TS** ; **dF = −SdT − PdV**]
{This thermodynamic potential is known as the Helmholtz free energy.}

10. *Continued:* Similarly, if the temperature T and pressure P are taken to be the independent thermodynamic variables, the appropriate thermodynamic potential (G) that is at a minimum in thermal equilibrium is ___. Its differential is given by ___.
[**G ≡ U + PV − TS** ; **dG = −SdT + VdP**]
{This thermodynamic potential is known as the Gibbs free energy.}